Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

sam.nitk.ac.in sam@nitk.edu.in

Computational Mathematics - MA 608
Problem Sheet - 4

Numerical Integration

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1. $I = \int_1^3 \frac{dx}{x}$ is evlauated by trapezoidal rule with 8 strips. Estimate the error in the value of I.
2. Evaluate
$$\int_0^6 \frac{dx}{1+x^2}$$
by using
   (a) Trapezoidal rule
   (b) Simpson's 1/3-rule
   (c) Simpson's 3/8-rule Weddle's rule

   and compare the results with its actual value.
3. Evaluate
$$\int_0^1 \frac{x^2}{1+x^2} dx$$
by using Simpson's 1/3- rule. Compare the error with the exact value.
4. Use the Trapezoidal rule to estimate the integral
$$\int_0^2 e^{x^2} dx$$
taking 10 sub-intervals.
5. Use Simpson's 1/3-rule to find
$$\int_0^{0.6} e^{-x^2} dx$$
by taking seven ordinates. Compare the approximate with the exact value.
6. Using Simpson's 3/8-th rule, compute the value of
$$\int_{0.2}^{1.4} (\sin x - \log x + e^x) dx.$$

7. The velocity v(km/min) of a moped which starts from rest, in given at fixed intervals of time t (min) as follows

$t:$	2	4	6	8	10	12	14	16	18	20
$v:$	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes.

8. The velocity v of a particle at distance s from a point on its linear path is given by the following table:

$s(m):$	0	2.5	5.0	7.5	10	12.5	15	17.5	20
$v(m/sec):$	16	19	21	22	20	17	13	17	9

Estimate the time taken by the particle to traverse the distance of 20 meters, using Boole's value.

9. A solid of revolution is formed by rotating about the x- axis, the area between the x- axis, the lines $x = 0$ and $x = 1$ and a curve through the points with the following co-ordinates.

$x:$	0	0.25	0.5	0.75	1
$y:$	1	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed using Simpson's rule.

10. A river is 80 ft. wide. The depth d in feet at a distance x ft. from one bank is given by the following table. Find approximately the area of the cross-section.

$x:$	0	10	20	30	40	50	60	70	80
$y:$	0	4	7	9	12	15	14	8	3

11. A body is in the form of a solid of revolution. The diameter D is cm. of its sections at distances x cm. from on end are given below. Estimate the volume of the solid.

$x:$	0	2.5	5	7.5	10	12.5	15
$D:$	5	5.5	6	6.75	6.25	5.5	4

12. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's 1/3-rd rule, find the velocity of the rocket at $t = 80$ seconds.

$t(sec):$	6	10	20	30	40	50	60	70	80
$f(cm/sec^2)$	30	31.63	33.34	35.47	37.75	40.33	43.23	46.69	50.67

13. Derive composite Simpson's 1/3-rule.

14. Derive composite Simpson's 3/8-rule.

15. Using composite Trapezoidal rule, evaluate

$$\mathtt{I} = \int_1^2 \int_1^2 \frac{dx\, dy}{xy}$$

taking four subintervals in each direction.

16. Apply composite Simpson's 1/3-rule to evaluate the integral

$$\mathtt{I} = \int_0^1 \int_0^1 xe^y dx\, dy, (h = k = 0.5)$$

17. Evaluate $\int_0^1 \int_0^1 (x + y) dxdy$ using Simpson's 1/3 rule with $h = k = 0.5$.

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